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**Submission date:** 23-Apr-2020 04:42PM (UTC+0700)

**Submission ID:** 1305408131

**File name:** 8\_How\_do\_open\_ended\_problems\_promote\_mathematical\_creativity.pdf (605.46K)

**Word count:** 3118

**Character count:** 17649

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## How do open-ended problems promote mathematical creativity? A reflection of bare mathematics problem and contextual problem

To cite this article: A Wijaya 2018 *J. Phys.: Conf. Ser.* **983** 012114

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# How do open-ended problems promote mathematical creativity? A reflection of bare mathematics problem and contextual problem

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**Abstract.** Creativity is often seen as one of the fundamental aspects of character education. As one of the 21st century skills, creativity has also been considered as an important goal of education across the world. This paper reports a study on promoting mathematical creativity through the use of open-ended mathematics problems. A total of 53 undergraduate students participated in the study. These students worked on open-ended problems in two types, i.e. bare mathematics problem and contextual problem. The contextual problem was presented in the form of paper-based and Geogebra-based. The students' works were analysed qualitatively in order to describe how students' mathematical creativity developed. It was found that the open-ended problems successfully promote students' creativity as indicated by various solutions or strategies that were used by students to solve the problems. The analysis of students' works show that students' creativity developed through three kinds of exploration, i.e. (1) exploration of contexts, (2) exploration of software features, and (3) exploration of mathematics concepts. The use of metacognitive questioning was found to be helpful to develop the first two explorations into mathematical exploration.

## 1. Introduction

Modern society that rapidly grows requires individual who has the ability to think, act, and communicate creatively [1, 2]. Consequently, creativity is seen as an important skill to deal with the demands of modern life, which later is called as 21st century skills. Considering its big role in society, creativity has long been placed as an essential goal of mathematics education [3–6]. Furthermore, there is evidence that mathematical creativity is positively correlated with mathematical ability [5]. This finding strengthens the argument for incorporating creativity in mathematics learning.

Robinson [2] defined creativity as an activity to produce original outcomes. Another definition of creativity is given by Torrance [7] who considered creativity as the capacity to identify gaps, obtain various solutions, and produce novel ideas. Torrance proposed four components of creativity, i.e. fluency, flexibility, originality, and elaboration. Fluency is an individual's ability to think of quantities of ideas. Flexibility refers to the ability to generate different categories of ideas. Originality is the ability to produce new ideas. Elaboration refers to the ability to elaborate ideas. In the 21st Century Skills Map for Mathematics [8], creativity is an ability to develop, implement, and communicate new ideas and diverse perspectives and ability to demonstrate originality. In mathematics, creativity is mainly connected to problem solving activities. Mathematical creativity is the process of generating unusual solutions and flexible problem solving skills [9, 10]. According to Eryvnyck [4] there are three levels of mathematical creativity. The first level is the preliminary technical stage which corresponds to "technical or practical application of mathematical rules and procedures, without the user having

any awareness of the theoretical foundation” (p. 42). The second level is the algorithmic activity which refers to performing mathematical techniques. In this second level students already consider theoretical foundation of the techniques they use, but they still emphasize on algorithmic or routine procedure. The third level is creative (conceptual, constructive) activity in which the mathematical activity involves non-algorithmic decision making.

In order to develop students’ creative thinking, mathematics learning should provide students with opportunity to freely try their own possible solutions [10]. With respect to this idea, many researchers suggested the use open-ended problems to stimulate students to divergent thinking and mathematical creativity. Kwon, Park, and Park [10] found students who worked on worksheets comprising open-ended problems had better divergent thinking including fluency, flexibility, and originality than their counterparts who worked on regular tasks in textbooks. Similarly, Levav-Waynberg and Leikin [11] who conducted a longitudinal study on mathematical creativity revealed that the use of multiple solution tasks could improve students’ fluency and flexibility. Another study on open-ended tasks was conducted by Wijaya [12]. In his study, Wijaya used exploratory task to investigate the modeling competence of prospective teachers. He found that exploratory tasks could lead the prospective teachers to figure out various strategies during their modeling process. Similarly, Wessels [13] also found that model-eliciting activities contribute to mathematical creativity.

Open-ended problems are often defined as problems that have multiple different solutions [14]. However, ‘solutions’ in this case are not restricted only to the results or answers of the problems. As mentioned by Becker and Epstein [15], an open-ended problem might have single solution which can be obtained by using many different ways. The key point of open-ended problems is that the problems give students opportunity and also stimulus to explore various solutions and/or ways or strategies. An open-ended problem can be given in the form of a bare mathematics task which means the task is presented in a mathematical form and symbol (see e.g. [15]) or a contextual problem, i.e. the task uses real world context (e.g. [12]).

The present study is aimed to explore students’ mathematical creativity when solving open-ended bare mathematics problem and open-ended contextual problem.

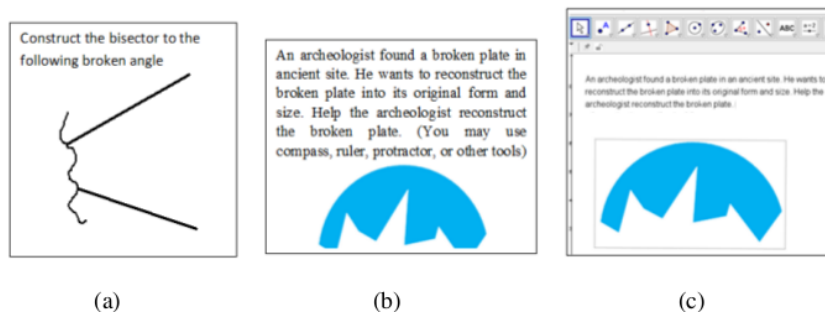
## **2. Methods**

### *2.1. Participants*

The participants of the study were two groups of undergraduate students enrolled in the Study Program of Mathematics Education which consisted of 21 third year students and 32 second year students. The reasons for selecting these participants were to promote their mathematical creativity and also to give them – as prospective teachers – insight about the importance of creativity in mathematics learning. The group of the third year students worked on the ‘broken angle problem’ (Figure 1.a), whereas the group of the second year students worked on the ‘broken plate problem’ (Figure 1.b and Figure 1.c).

### *2.2. Open-ended problems*

Two open-ended problems were used in the present study. The first problem was about constructing the angle bisector of a broken angle which was adopted from the work of Shimizu [15] (Figure 1.a). This problem was a bare mathematics problem, i.e. problem that was situated in an intra-mathematical context. The second problem was dealing with constructing a circle from a broken circle. This task was situated in extra-mathematical context which was also called as a contextual problem. Unlike the bare mathematics problem, the contextual problem required students to transform the contextual problem into a mathematical problem or to identify mathematics concept that was relevant to the problem. This contextual open-ended problem was given in two forms, i.e. paper-based (Figure 1.b) and Geogebra-based (Figure 1.c).



**Figure 1.** Open-ended bare mathematics problem and open-ended contextual mathematics problem

### 2.3. Data analysis

The data of this study was analysed qualitatively. For this purpose, the three levels of mathematical creativity that were proposed by Eryvncck [4] were used as the framework to analyse students' responses. The level 'preliminary stage' referred to students' works that did not include any mathematical strategies. The level 'algorithmic activity' was attached when students performed mathematical strategies without understanding the relevant mathematics concept(s). The last level, i.e. 'creative activity', was given to students who performed mathematical strategies with understanding the underlying mathematics concept(s).

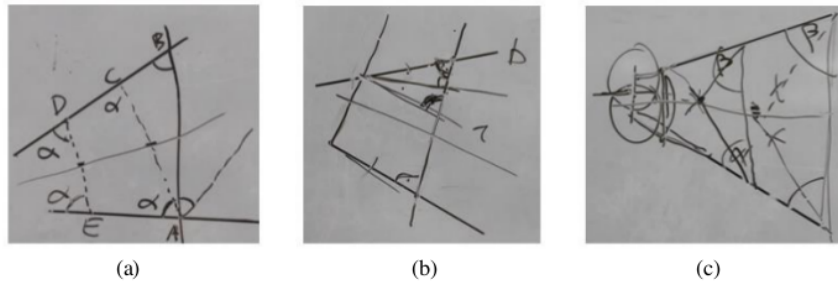
## 3. Result and discussion

### 3.1. Students' mathematical creativity when solving bare mathematics open-ended problem

Before students worked on the 'broken angle' problem, they were asked to construct the bisector of a 'complete angle'. All students drew a circle at the vertex that intersected the legs of the angle and constructing two congruent circles at these intersections. They said that there was no other strategy to construct an angle bisector. It seemed that the students were still 'trapped' in what they had learnt before so that they did not see other possible strategies to construct an angle bisector. In the next activity, students worked on the 'broken angle' problem. The students were not allowed to change the broken angle into a complete angle, i.e. by extending the legs of the broken angle. All of students agreed that they could not use the same method which previously was used to construct the bisector of the complete angle. After struggling about 45 minutes, the students came up with new strategies to construct an angle bisector. There were three different methods observed from students' works. The first strategy was using the idea that the bisector of the vertex angle of an isosceles triangle is the altitude to the base. Students who used this method constructed a segment intersecting the legs of the broken angle (see segment AB in Figure 2.a). In the next step, they drew segment AC to construct two congruent angles, i.e.  $\alpha$ , which each measured  $\frac{1}{2}(m\angle A + m\angle B)$ . Consequently, the segment AC became the base of an isosceles triangle. The students constructed the segment DE that was parallel to AC and constructed the perpendicular bisector to segment AC and DE. This perpendicular bisector was an altitude of the isosceles triangle and therefore was also the angle bisector of the vertex angle, i.e. the broken angle. The second strategy dealt with the idea of translation. Students who used this method translated a leg of the broken angle until it intersected the other leg so that a complete angle was constructed (see Figure 2b). This new complete angle was the measure as the 'broken angle'. These students constructed the angle bisector of the new complete angle. Their final step was translated this angle bisector to the opposite direction of the first translation and the distance of this second translation was a half of the distance of the first translation. This translated angle bisector was the angle bisector of the broken angle. The third strategy considered the idea that the angle bisectors of a triangle are concurrent. In this strategy students first drew two parallel segments that intersected the legs of the broken angle (see Figure 2.c). They constructed the angle bisectors of the four new



constructed angles. The last step was constructing a line connecting the intersection points of the two pairs of new angles. This line was the bisector of the broken line. This result indicates that the students already at the level of Eryvnyck's creative activity. Note: the students did not use proper tool and it seemed that they preferred to focus on the mathematical concept underlying the geometrical construction rather on the construction itself.



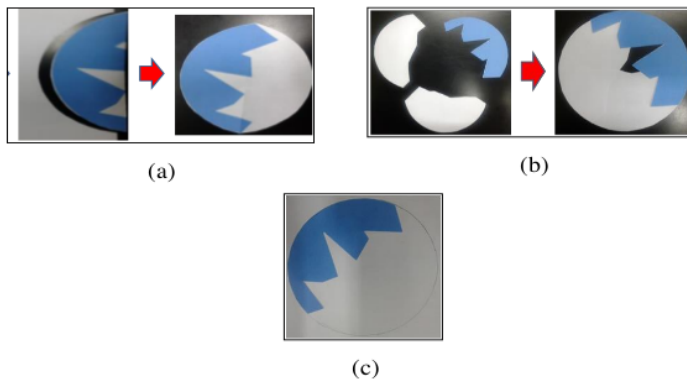
**Figure 2.** Students' new strategies to construct an angle bisector

### 3.2. Students' mathematical creativity when solving contextual open-ended problem

Unlike the case of the bare mathematics open-ended problem that directly led to mathematical exploration, for the contextual open-ended problem students did not directly explore mathematics concepts. All students knew that reconstructing the broken plate meant drawing a full circle, which indicated that the students were able to interpret the problem situations into mathematical terms. However, in the process of solving the problem several non-mathematical strategies emerged.

#### 3.2.1. The case of paper-based task

At the beginning of solving the problem, almost all students did hands-on activities without any mathematical notion. The students also did not really use 'mathematical tools', such as compass and ruler. Instead, they used scissors to cut the figure of the broken plate. There were three non-mathematical strategies that were used by the prospective teachers, i.e. 'fold-and-cut', 'duplicate-and-tessellate', and 'cut-and-rotate'. 'Fold-and-cut' strategy meant that students folded the figure of the broken plate and cut the paper along the curve or arc of the plate (Figure 3.a). For the second strategy, i.e. 'duplicate-and-tessellate', students made duplicates of the broken plate and arrange them to form a circle (Figure 3.b). Lastly, the idea of the 'cut-and-rotate' strategy was similar to the 'duplicate-and-tessellate' strategy. For this method the students cut the broken plate and rotated it in such a way to construct a circle (Figure 3.c). These strategies indicate the first level of mathematical creativity.



**Figure 3.** Students' non-mathematical strategies when solving the paper-based task

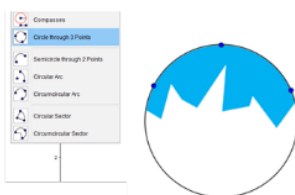
In their first attempt, none of the students used mathematical strategies. Therefore, a metacognitive question – ‘can you do it without cutting the broken plate?’ – was used to stimulate students to explore mathematical strategies. This question seemed to work well as it was seen from students’ mathematical strategies, i.e. using the intersection of the perpendicular bisectors of chords.



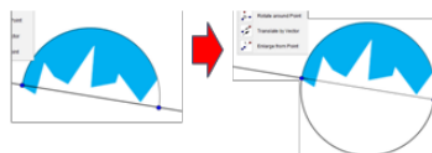
**Figure 4.** Students’ mathematical strategies

### 3.2.2. The case of Geogebra-based task

The students who got this problem directly explored the features of Geogebra. Most of students successfully constructed a circle by using the features of Geogebra. Figure 5 is an example of students’ work that was constructed by the feature ‘circle through three points’. This feature was quite clear, the students just simply plotted three points along the arc of the broken circle and a circle was automatically constructed. Figure 6 shows an example of circle which was constructed by using a combination of two features, i.e. ‘semicircle through 2 points’ and ‘reflect object in a line’. The students first constructed a semicircle and then reflect it through a line to obtain a circle.

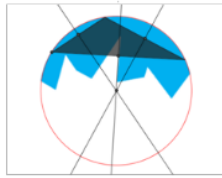


**Figure 5.** ‘Circle through three points’ strategy



**Figure 6.** A combination of semicircle and reflection strategy

Although these students successfully constructed a circle, from a discussion it was revealed that they were not aware about the mathematical ideas or concepts underlying their strategy. It means that the students are at the second level of Ervynck’s mathematical creativity: algorithmic activity. In order to trigger them to think about mathematical ideas, a metacognitive question – ‘what would you do if you use compass, instead of Geogebra’ – was posed. This question successfully led students to explore the mathematical ideas behind Geogebra’s features. Most students focused on investigating the ‘circle through three points’ feature. Some students found the idea of circumscribe triangle (see Figure 7). They argued that the three points represented the vertices of a triangle. Another mathematical idea given by students was constructing perpendicular bisectors of two chords.



**Figure 7.** A circumscribed-circle strategy

#### 4. Conclusion

In general, the results of the present study confirm other studies (e.g. [10– 12]) that open-ended tasks could promote students' mathematical creativity. Furthermore, in agreement with Levenson [6] this study also reveals that the presentation of the problem – whether a bare mathematics or contextual and whether a paper-based or Geogebra-based – leads to different types of exploration and different levels of mathematical creativity. The paper-based contextual problem could direct students to exploration the contexts of the problem and to explore the first level of mathematical creativity – preliminary stage – at which students used non-mathematical strategies. The Geogebra-based contextual problem led students' toward exploration of software features. In this exploration, students mainly focused on the features without substantial awareness of mathematical concepts behind the features which means they were at the second level of mathematical creativity, i.e. algorithmic activity. For these two forms of contextual problem, the third level of mathematical creativity – i.e. creative activity – and mathematical exploration were achieved after the use of metacognitive questions. The bare mathematics problem directly led to mathematical exploration and to the level of creative activity. To conclude, the present study suggests a combination of various forms of open-ended problems and metacognitive questions in order to develop students' mathematical creativity.

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